

# Spectral cleaning in MST Radar Data using the Modified Cepstrum

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**Abstract**— The concept of cepstrum thresholding (CT) is applied to estimate smoothed nonparametric spectrum. The CT method is tested for various simulated signals such as broad, narrow band and two-tone sinusoidal signals to estimate spectrum. The CT method works well in detection of peaks to a maximum signal to noise ratio (SNR) of -20dB. To improve SNR, the CT technique needs to be modified. The modified cepstrum (MC) approach is developed and implemented, to validate, it has been applied to MST radar data. It is found that proposed method works well in variance reduction even at higher altitudes and results are compared with the normal CT method and the periodogram.

**Index Terms**— Cepstral thresholding, Doppler profile, Fast Fourier Transform, Periodogram, Signal to Noise ratio, Smoothing window, and Spectral cleaning.

## 1 INTRODUCTION

National Atmospheric Research Laboratory (NARL) at Gadanki (13.47°N, 79.18°E), India has been operating 53 MHz atmospheric Mesosphere, Stratosphere and Troposphere (MST) radar for studying structure and dynamics of lower, middle and upper atmosphere [1]. MST Radar provides estimates of atmospheric winds on a continuous basis with high temporal and spatial resolutions. MST Radar uses the echoes obtained over the height range of 1-100 Km to study winds, turbulence [2],[3],[4]. The Indian MST Radar is operational for scientific studies of the atmosphere in the height range of 2-20 km (troposphere and lower stratosphere), 60-90 km (mesosphere), 100-150 km (E region) and 150-800 km (F region). The echoes from the atmosphere are due to neutral turbulence in the lower height regions and due to the irregularities in electron density in higher altitudes.

In this paper, an attempt has been made to apply the concept of Cepstrum Thresholding (CT) for smoothed spectrum estimation [5]. To achieve additional variance reduction, the CT method is modified by weighting the cepstral coefficients with the smoothing window. The Modified Cepstrum (MC) approach has been applied to the MST radar data.

The theory of the smoothed spectrum estimation using the MC has been presented in section 2. A stepwise description of the algorithm applied to MST radar data is given in section 3. The results and discussion are in section 4 and the conclusions are in section 5.

## 2 SMOOTHED SPECTRUM ESTIMATION VIA THE MODIFIED CEPSTRUM

Consider a stationary, discrete-time, signal  $x(n), n = 0, 1, 2, \dots$  with covariance sequence  $r_k, k = -\infty$  and

power spectral density (or spectrum)  $\phi_p(\omega)$  where  $\omega \in [-\pi, \pi]$ . In practice, the spectrum  $\phi(\omega)$  is estimated from a set of observed samples  $x(n)_{n=0}^{N-1}$  of the signal. The periodogram estimate of  $\phi(\omega)$  is given by [6],[7],[8]

$$\hat{\phi}_p(\omega) = \frac{1}{N} \left| \sum_{n=0}^{N-1} x(n) e^{-i\omega n} \right|^2 \quad (1)$$

where the subscript 'p' denotes the Periodogram estimate.

Let

$$\omega_l = \frac{2\pi}{N} l, l = 0, \dots, N-1 \quad (2)$$

denote the Fourier grid of the angular frequency axis.  $\hat{\phi}_p(\omega)$  can be computed efficiently by means of a Fast Fourier transform (FFT) algorithm.

The Cepstral coefficients are defined as

$$c_k = \frac{1}{N} \sum_{l=0}^{N-1} \ln[\phi(\omega_l)] e^{j\omega_l k}, k = 0, 1, \dots, N-1 \quad (3)$$

where it is assumed that  $\phi(\omega_l) > 0, \forall l$ . For real valued signal, the cepstral coefficients have several interesting features, one of which is mirror symmetry:

$$c_{N-k} = c_k, k = 0, 1, \dots, N/2 \quad (4)$$

which mean that only half of the sequence  $c_0, \dots, c_{N/2}$ , is distinct. The other half is obtained from  $c_1, \dots, c_{(N/2)-1}$ , as in (4). As in case of complex valued signal, the cepstral coefficients are complex conjugate pair. The spectral values do not follow the property of even symmetry.

Using the periodogram estimate in (1), a common estimate of the cepstral coefficients is obtained by replacing  $\phi(\omega)$  in (3) with  $\hat{\phi}_p(\omega)$ , from [5], [9].

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$$\hat{c}_k = \frac{1}{N} \sum_{l=0}^{N-1} \ln[\hat{\phi}_p(\omega_l)] e^{j\omega_l k} + \gamma \delta_{k,0}, \quad (5)$$

$$k = 0, \dots, M,$$

$$\text{where } \delta_{k,0} = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{else} \end{cases} \quad (6)$$

$M=N/2$  and  $\gamma = 0.577216\dots$  (the Euler's constant).

It was shown that in large samples, the estimated cepstral coefficients  $\{\hat{c}_k\}_{k=0}^M$  are independent normally distributed random variables [11]:

$$\hat{c}_k \sim N(c_k, s_k^2) \quad (7)$$

$$\text{with } s_k^2 = \begin{cases} \frac{\pi^2}{3N} & \text{if } k = 0, \dots, M \\ \frac{\pi^2}{6N} & \text{if } k = 1, \dots, M-1 \end{cases} \quad (8)$$

The Cepstral coefficients are weighted by the smoothing window such as hamming or hanning window. The process of weighting is the lower indexed cepstral coefficients are same (more weight), but the higher indexed cepstral coefficients are modified (less weight). The modified cepstral coefficients are denoted by  $\tilde{c}_k$ . The smoothed spectral estimate corresponding to  $\tilde{c}_k$  is given by:

$$\tilde{\phi}_{cep}(\omega_l) = \exp \left[ \sum_{k=0}^{N-1} \tilde{c}_k e^{-j\omega_l k} \right]; \quad l = 0, \dots, N-1 \quad (9)$$

The final scaled spectrum estimate  $\hat{\phi}_{cep}(\omega_l)$  is then given by

$$\hat{\phi}_{cep}(\omega_l) = \hat{\alpha} \tilde{\phi}_{cep}(\omega_l), l = 0, \dots, N-1 \quad (10)$$

$$\text{where } \hat{\alpha} = \frac{\sum_{l=0}^{N-1} \hat{\phi}_p(\omega_l) \tilde{\phi}_{cep}(\omega_l)}{\sum_{l=0}^{N-1} \tilde{\phi}_{cep}^2(\omega_l)} \quad (11)$$

### 3. EXISING AND PROPOSED METHODS

#### 3.1 Periodogram

The MST radar raw data (I and Q) has been used and then the coherent integration was performed. It improves the process gain by a factor of inter pulse period (IPP) and also improves the SNR by integrating the detected quadrature due to any other succeeding operation. The basic Periodogram method is used to find the spectral contents of a radar signal. This method is not consistent (high variance) because of erratic fluctuations in its spectrum.

#### 3.2 The Proposed Method

The consistent method to reduce the variance estimate is the CT approach. Stoica and Sandgren (2006) proposed the CT method for the smoothed spectrum estimation. Practically, the variance reduction in spectrum estimation using the CT method is not significant. To achieve better variance reduction, the CT method is modified, called as the MC method. To

check the validity, the MC approach is applied to MST radar data.

The algorithm steps for the MC method are as follows.

- Step1: Coherent integration is performed on raw data
- Step2: Compute power spectrum using FFT
- Step3: Determine the cepstrum from the power spectrum
- Step4: Multiply the cepstral coefficients by the smoothing window
- Step5: Determine the power spectrum from the cepstral coefficients selected.

### 4 RESULTS AND DISUSCUSSION

The Periodogram, the CT and The MC techniques are applied to 5<sup>th</sup>, 30<sup>th</sup>, 55<sup>th</sup>, 80<sup>th</sup> and 105<sup>th</sup> bins of East beam radar data for spectral estimation. From Figs. 1, 2 and 3, it is observed that similar spectral contents produced by the Periodogram and CT methods. The results confirm that the estimated spectra based on MC technique are noise free, smoothed envelope when compared with the above methods. It enhances the ability to extract original Doppler frequency from the Doppler profiles. The MC approach was tested for all six beams of MST radar data. It was observed that the MC approach produces the smoothed spectrum estimation even at higher altitudes.

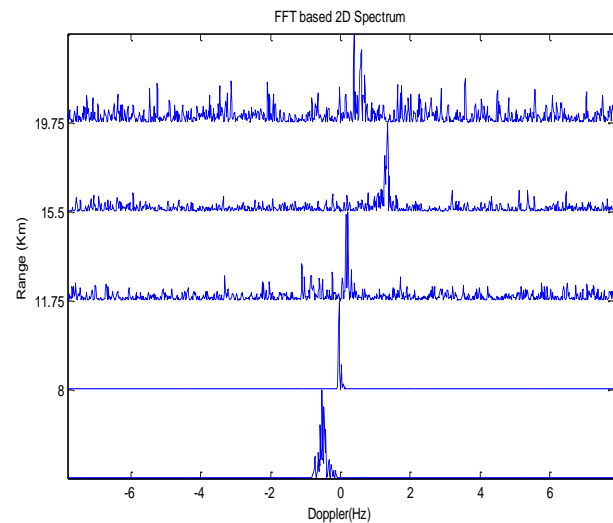


Fig. 1 FFT based Spectra of East beam data at various heights.

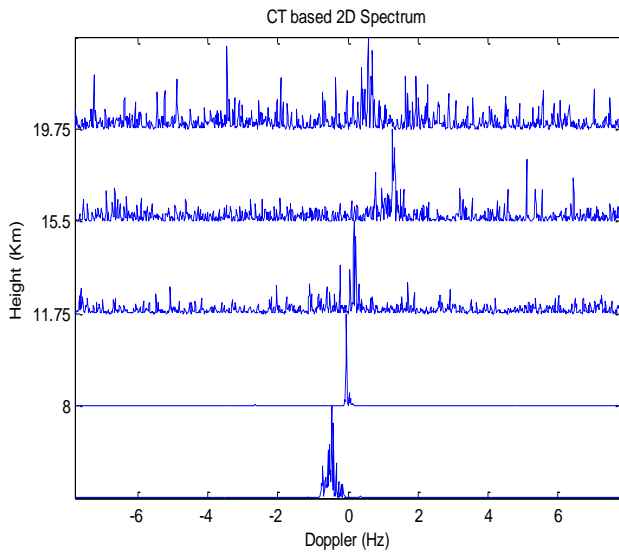


Fig. 2 CT based Spectra of East beam data at various heights.

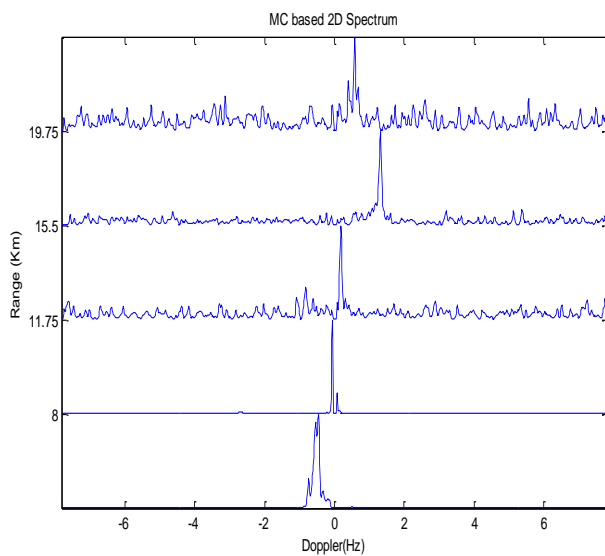


Fig. 3 MC based Spectra of East beam data at various heights.

## 5 CONCLUSIONS

The CT method has been applied to MST radar data for the smoothed spectrum estimation. The CT method fails to achieve better spectral cleaning compared with the conventional technique of Periodogram. The MC method is developed and implemented for the radar data. After testing with the six beams of radar data, the MC approach is appropriate to clean the spectral contents at the expense of computational complexity. To reduce the computational burden, one can replace the cepstrum by the discrete cepstrum.

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